NUMERICAL STUDY OF TEMPERATURE FIELDS IN THE CONTINUOUS TEEMING OF STEEL INGOTS OF RECTANGULAR CROSS SECTION

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The thermal boundary layer approximation is used to examine the solidification of steel ingots of rectangular cross section during continuous teeming and the effect of the sides on solidification time.

The structure of a solidifying ingot is heavily influenced by its shape and dimensions [1]. A substantial amount of relevant empirical data has now been accumulated and has been augmented by data obtained through analog and digital modeling. In particular, there has been broad speculation as to the effect of the ratio of the sides of a rectangular ingot on solidification time (see [2], for example). However, the data from full-scale tests and from calculations agree only qualitatively. To obtain quantitative agreement as well, it will be necessary to significantly improve the accuracy of the full-scale measurements, refine the mathematical models of teeming processes, and use better computational algorithms to study these processes.

This study is a continuation of [3], which examined the dependence of the solidification process on different (physical and geometric) parameters in the continuous teeming of cylindrical steel ingots. We will employ the well-known thermal boundary layer model [4]. In modeling rectangular ingots, this model allows us to change over from a three-dimensional quasisteady Stefan problem to a two-dimensional "nonsteady" problem for a parabolic equation. Further simplification of the problem (without loss of accuracy) entails the construction of a unidimensional model which ignores the dependence of solidification on the cross section of the ingot with respect to its variation along the large side. Within the framework of this approximation, it is possible to study various interesting mathematical models of a continuous-cast ingot. For example, it is possible to allow for the motion of the melt. The present investigation examines the question of the ratios of the sides of a rectangular ingot for which it is possible to use a unidimensional approximation. To solve two-dimensional heat-conduction problems with phase transformations, we developed an algorithm based on the most rapidly converging iteration methods currently available for solving grid problems.

Mathematical Model. We direct the OZ axis along ingot being withdrawn from the center. The ingot has a rectangular cross section $G = \{(x, y) \mid |x| < a, |y| < b\}$. The temperature field inside the ingot is described by the heat-conduction equation:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)=cv\frac{\partial T}{\partial z}, \quad (x, y)\in G, \quad 0<z<\infty. \tag{1}$$

We will assume that the thermophysical characteristics of the entire ingot are constant. The Stefan conditions below are satisfied at the phase boundary S, where $T = T^*$:

$$T_1 = T_2 = T^*, \quad (x, y, z)\in S, \tag{2}$$

$$\left(k\frac{\partial T}{\partial n}\right)_1-\left(k\frac{\partial T}{\partial n}\right)_2=-\lambda v\cos(n, z), \quad (x, y, z)\in S. \tag{3}$$

In (2) and (3), the subscript 1 corresponds to the region of the melt G_1, where $T > T^*$ (liquid phase). The subscript 2 corresponds to the hardened ingot G_2, where $T < T^*$ (solid phase). The letter n denotes an outward (in relation to G_1) normal to S, while cos (n, z) represents the cosine of the angle between the normal n and the axis OZ.

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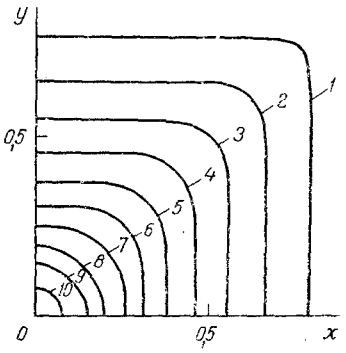

Fig. 1

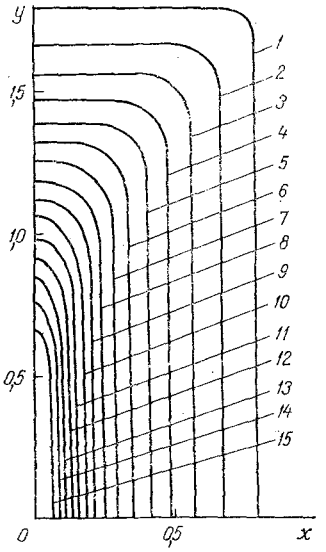

Fig. 2

Fig. 1. Fusion isotherms for an ingot of square cross section with $u_0 = 1.02$, $u_\alpha = 0.2$, Pe = 150, St = 0.2, $Bi_0 = 3.0$, $\varkappa = 0.0$, at $z = i\Delta z$, $i = 1, 2, \ldots, 10$, $\Delta z = 9.375$.

Fig. 2. Phase boundary for the variant with $u_0 = 1.02$, $u_\alpha = 0.2$, Pe = 150, St = 0.2, $Bi_0 = 3.0$; $Bi_1 = 0.2$, $\varkappa = 0.01$, $\mu = 2$, at $z = i\Delta z$, $i = 1, 2, \ldots, 15$, $\Delta z = 9.375$.

The Stefan approximation (2)-(3) may be incorrect for modeling the solidification of steel ingots. Here, it is preferable to use different models of a two-phase region [5]. As an example, along with the Stefan approximation, we studied the teeming of steel in ingot molds and on a continuous caster through the use of a model of an equilibrium two-phase region. In this model, solidification occurs in the solidus–liquidus temperature range. The location of this range depends on the grade of steel, and the range is fairly small for low-carbon steels. Also, as calculations show, the width of the two-phase zone does not significantly affect the result. Thus, we restricted ourselves to (2) and (3) in studying the effect of the form of the cross section of a steel ingot on the solidification process.

Let us discuss the boundary conditions for Eq. (1). At z = 0, the temperature of the ingot coincides with the temperature of the molten metal being teemed, i.e.,

$$T(x, y, 0) = T_0, \quad (x, y) \in G. \tag{4}$$

With sufficiently large z, the ingot cools to the ambient temperature:

$$T(x, y, z) \to T_\alpha, \quad z \to \infty, \quad (x, y) \in G. \tag{5}$$

The heat transfer on the lateral faces of the ingot is given by the relation

$$k \frac{\partial T}{\partial n} = -\alpha (T - T_\alpha), \ (x, y) \in G, \quad 0 < z < \infty, \tag{6}$$

where $\alpha = \alpha(z)$ is an assigned function.

We will use the thermal boundary layer approximation [5] to solve boundary-value problem (1)-(6). This approximation is valid for sufficiently high withdrawal velocities v.

After changing Eqs. (1)-(6) to dimensionless form, we obtain:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \mathrm{Pe} \frac{\partial u}{\partial z}, \quad (x, y) \in G, \quad z > 0, \tag{7}$$

$$(u)_1 = (u)_2 = 1, \quad (x, y, z) \in S, \tag{8}$$

$$\left(\frac{\partial u}{\partial n}\right)_1 - \left(\frac{\partial u}{\partial n}\right)_2 = -\mathrm{Pe\,St} \cos(n, z), \quad (x, y, z) \in S, \tag{9}$$

$$u(x, y, 0) = u_0, \quad (x, y) \in G, \tag{10}$$

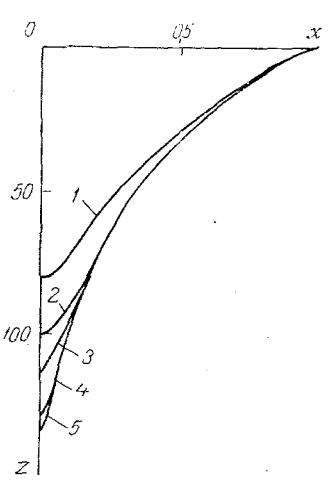

Fig. 3

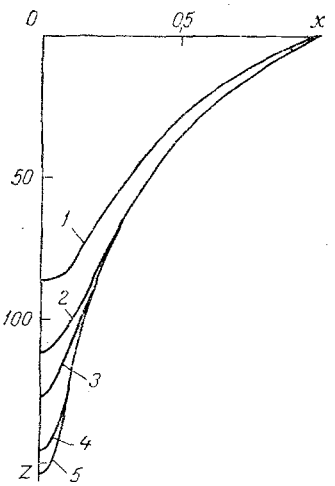

Fig. 4

Fig. 3. Section of the crystallization front at y = 0 (u_0 = 1.02, u_a = 0.2, Pe = 150; St = 0.2, Bi_0 = 3.0, $\varkappa$ = 0.0): 1) cylindrically symmetrical ingot; 2) μ = 1; 3) 1.2; 4) 1.6; 5) unidimensional ($\mu = \infty$).

Fig. 4. Effect of the boundary regime on the form of the crystallization front (u_0 = 1.02, u_a = 0.2, Pe = 150, St = 0.2, Bi_0 = 3.0, Bi_1 = 0.2, $\varkappa$ = 0.01): 1) cylindrical case; 2) μ = 1; 3) 1.2; 4) 2.0; 5) unidimensional ($\mu = \infty$).

$$\frac{\partial u}{\partial n} = -\mathrm{Bi}(u - u_a), \quad (x, y) \in \partial G, \quad z > 0, \tag{11}$$

where $G = \{(x, y) \mid |x| < 1, |y| < \mu\}$.

As in [4], we used the following relation for the lateral cooling regimes:

$$\mathrm{Bi}(z) = (\mathrm{Bi}_0 - \mathrm{Bi}_1)\exp(-\varkappa z) + \mathrm{Bi}_1, \tag{12}$$

where $Bi(0) = Bi_0$, $Bi(\infty) = Bi_1$.

Finite-Difference Algorithm. Two main approaches are used to numerically solve problems of the Stefan type. The first involves determination of the unknown phase boundary (see [6], for example). The second approach, based on the use of a generalized formulation of the problem, makes it possible [7] to readily construct through-type difference schemes for solving multidimensional grid problems. We used the second approach here and we smoothed the coefficients. Let us briefly discuss certain features of the algorithm being employed.

Equation (7) can be written with the Stefan conditions (8), (9) as a single equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \mathrm{Pe}(1 + \mathrm{St}\,\delta(u - 1))\frac{\partial u}{\partial z}, \quad (x, y) \in G, \ z > 0. \tag{13}$$

In an approximate solution of problem (13), (10), (11), the δ-function in the right side of (13) is replaced by a smoothed function δ_Δ. For example:

$$\delta_\Delta(u - 1) = \begin{cases} \dfrac{1}{2\Delta}, & |u - 1| \leqslant \Delta, \\ 0, & |u - 1| > \Delta. \end{cases}$$

The temperature of the melt being teemed u_0 is close to the phase transformation temperature u^*, i.e., problem (7)-(11) is close to being a one-phase problem. As shown by numerical experiments, the use of a standard smoothing scheme of the type (14) results in fairly large errors. For such problems, it is convenient to use the procedure of "local" smoothing of the δ-function. In the unidimensional case, this corresponds to the use of (14) with $\Delta = \Delta(x)$, i.e., the "spreading" interval depends on the point x [8]. We used a similar "local" smoothing procedure in the solution of problem (7)-(10).

We employed an implicit difference scheme to approximate the differential problem. The difference problem was solved by an iterative process which successively refined the unknown

boundary. The elliptical grid problem was solved on the basis of the ELLDEC application package [9]. We used the MAFCG2 subroutine, which realizes an iterative method involving approximate factorization [10]. We should point out that, for this method, the rate of convergence is independent of the coefficient with $\partial u/\partial z$. The authors of [11], using a somewhat larger memory, obtained similar results by an alternate-triangular iteration method. These methods of inverting the elliptical grid operator on the upper level make it possible to construct purely implicit schemes having characteristics similar to more economical schemes.

Sample Calculations. We will present some results of calculations performed to solve problem (7)-(11). The results serve to augment the data in [3]. For the variant with $u_0 = 1.02$, $u_a = 0.2$, Pe = 150, St = 0.2, $Bi_0 = 3.0$, $\varkappa = 0.0$, Fig. 1 shows the phase-boundary lines u = 1 with different z in the case of an ingot having a rectangular cross section. The calculations were performed for one-fourth of G on a uniform 41 × 41 rectangular grid. The results of similar calculations with $\mu = 2$ are shown in Fig. 2. It is evident that the solidification time (depth) for such an ingot cross section is actually determined by the time of solidification of the central section. Figure 3 shows more accurate quantitative dependences of solidification time on the elongation of the rectangular ingot μ. The same figure also shows data for a cylindrically symmetrical ingot (y = 0). The effect of the boundary regime ($\varkappa = 0.01$, $Bi_0 = 3$, $Bi_1 = 0.2$) is shown in Fig. 4.

The results of the calculations show that at $\mu \geq 2$, the effect of ingot elongation on solidification time (depth) cannot be ignored within the range of parameters investigated here.

NOTATION

(x, y, z), cartesian coordinates; v, ingot withdrawal velocity; T*, phase transformation temperature; T_a, ambient temperature; T_0, initial temperature of the melt; λ, heat of phase transformation; k, thermal conductivity; c, specific heat; α, coefficient of heat transfer with the environment; Pe = vac/k, Peclet number; St = λ/cT^*, Stefan number; Bi = $\alpha a/k$, Biot number.

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